Investment Financing And The Moral Hazard Index

A.J. Aisha Nuddin[1] and A.K.M. Azhar[2*]

ABSTRACT

This paper proposes a new approach to the analysis and measurement of moral hazard in investment financing. In context of a financial contract, we construct the moral hazard index to measure the tolerable threshold of the moral hazard risk subject to the entrepreneur's equity share, project capital, expected profits and debt (if exists). We also investigate the differences in debt and equity contracts in the context of default probabilities subject to the factors encompassed in the moral hazard index. We generalize the boundary conditions of this index in quantifying the cut-off level of the moral hazard risk faced by banks in both profit loss sharing and mark-up based types of contracts. This neew moral hazard index methodology complements existing credit risk ranking of customers used by financial institutions in particular Islamic banks.

1.0 INTRODUCTION

Credit risk is the risk of loss face by a financial institution (FI) (lender) when customer (debtor) is unable to pay its loan obligation as laid out in a legal contract. Sophisticated screening procedures are now being implemented to measure and analyse customers credit risk before a financial contract is finalised. Some FIs establish departments to assess and manage the credit risk ratings of their customers. Some use the services of established credit risk assessing companies to perform the job for a specified amount of fee. Most FIs apply their own models (credit scorecards) to assess their customers in accordance with their risk undertakings.. Some require collateral to secure the loan repayment for certain of their products. The significant of these issues for the survival and prosperity of financial institutions have lead to credit risk modeling turn into an important issue in both academic and banking communities. (Lean Yu et al, 2008).

1 Universiti Teknologi MARA, Arau 02600 Perlis, Malaysia and INSPEM, Universiti Putra Malaysia.

2* Correspondence: email: akmazhar@upm.edu.my, Fax: +603 8946 6188. Department of Accounting and Finance, IKDPN and INSPEM, Universiti Putra Malaysia 43400, UPM Serdang Selangor, Malaysia.

Altman and Saunders (1998) presented a classical overview on credit risk modeling over the past 20 years stated that a subjective analysis system is the dominant form of system in which banks used to assess the credit risk of corporate loans applications. This system used the four "Cs" of credit respectively i) character, debtor reputation or performance record ii) capital, debtor capital leverage iii) capacity, liquidity of debtor asset (cash, bank, stock etc.) and iv) collateral, types of collateral presented by debtor. Saunders and Allen (2002) called it the five "Cs" of credit with an additional C for economic cycle.

Nowadays FIs are moving away from the above system towards more sophisticated techniques. Chengcheng Hao et al. (2010) summarised the developments of credit risk modeling from January 1998 to April 2009, to which they found that the focus of research relating to credit risk modeling have moved away from static individual level models to dynamic portfolio models.

The four widely used credit risk analysis techniques are the external rating services, financial statement analysis models, structural model and the transition models of Credit Metrics and Credit Portfolio View (Allen and Powell, 2011). The external rating services analyse the credit risk by taking into account a wide range of factors such as environmental conditions, competitive position, management quality and the financial strength of the business. Financial statement analysis models provide a credit risk rating based on the analysis of financial statement items and accounting ratios of individual borrowers for the past several years such as measure of liquidity, activity ratio and leverage measures. Example of these models is the ZETA model developed by Altman (1968, 1977) which computes the probability of a firm going bankrupt within a two-year time period.

Structural models measure changes to default probabilities by comparing a firm assets and its liability. Increase in the volatility of the assets will moves a firm closer to default (Allen and Powell, 2011). Under this model a default event is deemed to happen when its assets reach a sufficiently low level compared to its liabilities. This model provides a link between the credit quality of a firm and the firm's economic and financial conditions (Abel Elizalde, 2006). Examples of the structural models are the Merton's (1974) model and the Black and Cox model (1976). As for the transition models of Credit Metrics and Credit Portfolio View, they use historical data to form a transition matrix which show the probability of a borrower moving from one credit grade to another. Credit Portfolio View also uses a transition matrix approach but with different assumption concerning borrowers of the same grade (Allen and Powell, 2011).

This paper differ from the normal approach of credit risks analysis techniques discussed above. We propose a simple model in analysing credit risk from the morality perspective. Our attempt is to define and quantify the moral hazard risk in Islamic banking mudarabah (equity) and murabahah (mark up) loans based upon the entrepreneur utility function. We derive and propose an index to measure the tolerable threshold of the moral hazard risk subjected to entrepreneur's equity, project capital, expected profit and debt(if exists). The index is to help quantify the threshold or cutoff level of the moral risk face by a bank if an entrepreneur equity share for a project is , the capital amount is F, the expected profit amount is E and the amount of debt is D. This index is intended to complement the existing analysis in assessing credit risk on corporate loans.

2.0 MORAL HAZARD INDEX

2.1 Entrepreneur's Default Utility Function.

In this paper we consider modifying Aggarwal and Yousef (2000) entrepreneur utility function in defaulting from a project. The entrepreneur generates utility from diverting cash flow but utility generates in defaulting with certain amount of cash is not the same as the utility generates from receiving that amount through legal means.

Another way of saying this is that the satisfaction an entrepreneur receives in diverting x amount of cash flow illegally is not the same as the satisfaction he receives in obtaining x amount of cash flow through legal means.

The entrepreneur's utility from receiving a direct cash payment of from the firm (through legal means) is $V(x) = \frac{x}{n}$, $n \geq 1$, where n is the effort factor, if no effort is needed then $n = 1$.

The more the effort needed to receive a direct cash payment of x from the firm the less is the satisfaction of the entrepreneur. This mean bigger n implies more effort needed for a contract where $n = 1$ implies that the amount x will be received directly without any effort.

The satisfaction an entrepreneur received from diverting x amount of cash illegally is not the same as the satisfaction he received from obtaining direct cash from the firm legally. The satisfaction depends upon the entrepreneur level of morality consciousness. The entrepreneur's utility from diverting illegally an amount x is where $U(x) = cx$. where $0 \leq c \leq 1$. Here c is what we define as the level of moral hazard of an entrepreneur.

An entrepreneur with *c* equal to 0 is the one which will not default at any circumstances since an entrepreneur with this level of *c* hold on to a very high moral standard. The level of moral hazard of unity is the morality extreme where the entrepreneur is sure to default, the entrepreneur with this level of *c* is categorised as of the lowest moral standard.

2.2 Moral Hazard Index (MHI)

An entrepreneur who has zero wealth wants to run a business project. A bank agrees to finance the project base on profit lost sharing contract (mudarabah) or mark up contract (murabahah).

The entrepreneur is the "mudharib" who will run the project and the bank is the "rabbul maal" who will provide the capital for the project which we assume to be the same as the cost of the project.

Let E = Expected income of the project,

F = Amount of capital (Cost) provided by the bank.

E - F = expected profit

D = (Capital (cost) + mark up amount) = amount of debt the entrepreneur has to pay the investor.

a = entrepreneur's equity share.

c = cut-off level of moral hazard

n = the effort factor

In all, the contract is feasible only if the entrepreneur profit is greater than 0 that is when we have:

E - F - D > 0. (realistically we will only consider feasible contract)

For an entrepreneur not to default, his satisfaction in receiving the equity share from the expected profit minus the debt he should pay to the bank should be greater than his satisfaction (depend on c) of defaulting with the capital (provided by the bank).

Satisfaction in receiving the equity share minus debt can be written as:

$$\frac{\mathrm{a}\,(expected\ profit\ -\ debt)}{n} = \frac{\mathrm{a}\,(E - F - D)}{n}$$

Satisfaction of defaulting with the capital which depend on the his level of moral hazard can be written as *c(Captial) = cF*.

So for the entrepreneur not to default

$$\alpha \frac{(expected\ \ profit - D)}{n} > c(capital\ )$$

$$\alpha \frac{(E - F - D)}{n} > cF$$

$0 \le c < \dfrac{(E - F - D)}{nF}$ is the non default zone of the entrepreneur.

So $\alpha \dfrac{(E - F - D)}{nF} < c < 1$ is the default zone of the entrepreneur.

So $c = \alpha \dfrac{(E - F - D)}{nF}$ is the cut-off level of moral hazard which depend on

α, n, D, F and E.

This can be written as $\alpha = c\left(\dfrac{nF}{E - F - D}\right)$ ---------- (1)

There is no debt for pure equity contract *(mudarabah)* implies $D = 0$, so (1) can be simplified as $\alpha = c_e\left(\dfrac{nF}{E - F}\right)$. In the case of mark-up contract (murabahah) the entrepreneur gets all the profit so $\alpha = 1$ and (1) can be simplified as $1 = c_D\left(\dfrac{nF}{E - F - D}\right)$.

Here we use c_e as the cut-off level of moral hazard for *mudarabah* contract and c_D as the cut-off level of moral hazard for murabahah contract.

The cut-off level of moral hazard index for murabahah is $c_D = \dfrac{(E - F - D)}{nF}$, and the cut-off level of moral hazard index for mudarabah is $c_e = \alpha \dfrac{(E - F)}{nF}$.

For every set of α, *n, F, E and D* there is a c which is the cutoff level of moral hazard for the set, so any measure proportional to c can be taken as an index measuring the moral hazard corresponding to a particular set of α, *n, F, E and D.*

Let $\beta = \alpha(E - F - D)$

So $\beta = cnF$

Consider a plot of β *vs F,* the equation $\beta = cnF$ will be represented by rays passing through the origin with nc as the gradient of each ray.

For mudarabah contract we have D = 0, so $\beta = \alpha\ (E - F)$ and for murabahah contract, $\alpha = 1$ so $\beta = E - F - D$

Since $0 \le c \le 1$ then $nc_e, nc_D \in [\,0, n\,]$. The slope of the rays are nc ie. c is directly proportional to the slope.

$nc = \dfrac{\beta}{F}$, however this ratio is not suitable as an index since it is not proportionately scaled and also not symmetrical in the graph β versus F .

$\dfrac{\partial\left(\dfrac{\beta}{F}\right)}{\partial} = \dfrac{1}{F}$ and $\dfrac{\partial\left(\dfrac{\beta}{F}\right)}{\partial F} = -\dfrac{1}{F^2}$ imply that the ratio is not proportionately scaled.

If $f(\beta, F) = \dfrac{\beta}{F} = 1 - b$ then $f(F, \beta) \neq 1 + b$, this implies that the ratio is not symmetry about the diagonal of the graph β versus F as shown in figure 1

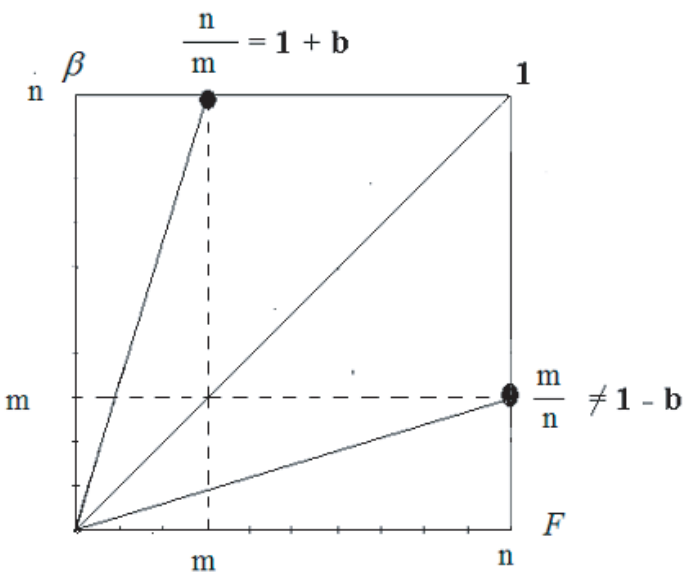

Figure 1 . Disproportionate Scaling and not symmetric

To overcome this problem we consider the use of the following transformation in the tradition of Azhar and Elliott (2006).

$$\text{MHI} = f(\beta, F) = 1 - \frac{\beta - F}{\beta + F} = \frac{(\beta + F) - (\beta - F)}{\beta + F} = \frac{2F}{\beta + F}$$

This index is proportionately scaled and symmetric as shown by the partial derivative below and figure 2.

$$\frac{\partial\left(\frac{2F}{\beta + F}\right)}{\partial\beta} = \frac{-2F}{(\beta + F)^2} \text{ and } \frac{\partial\left(\frac{2F}{\beta + F}\right)}{\partial F} = \frac{2}{(\beta + F)^2}$$

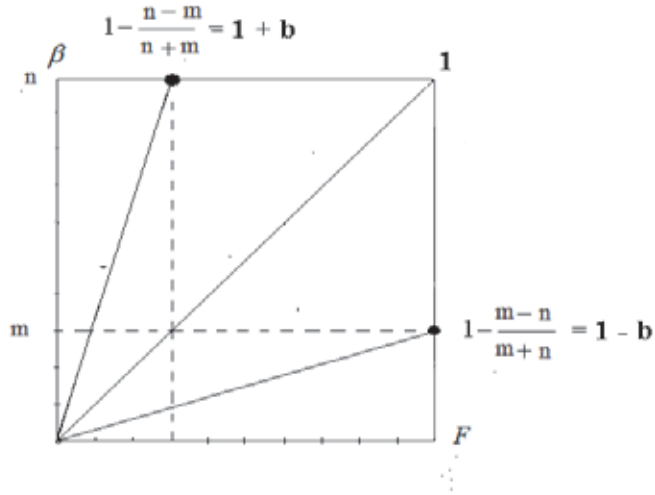

Figure 2. Proportionate scaling

$$\text{MHI} = \frac{2F}{\beta + F} = \frac{2\left(\frac{\beta}{nc}\right)}{\beta + \left(\frac{\beta}{nc}\right)} = \frac{\frac{2\beta}{nc}}{\frac{\beta(nc+1)}{nc}} = \frac{2}{1 + nc}$$

$$\frac{d\left[\frac{2}{1 + nc}\right]}{dc} = -\frac{2}{(1 + c)^2}$$ so MHI decreases as c increases . The following limits show that the range of the index is given by $0 \le \text{MHI} \le 2$.

$$\lim_{nc \to 0} \frac{2}{1 + nc} = 2 \quad \text{and} \quad \lim_{nc \to \infty} \frac{2}{1 + nc} = 0$$

Since [0,*nc*] is the interval for *nc* in which the entrepreneur does not default so we can observe as *nc* increases (decreases) the probability of the entrepreneur defaulting decreases (increases). Thus the higher the value of *nc*, the lower is the probability of default, *nc* is inversely proportional to default. Since the moral hazard index should measure the level of moral risk faced by the bank in which the entrepreneur will default, a measure which is inversely proportional to *nc* is needed as an indicator to the moral risk.

From the above description we can observe MHI is a suitable measure to be used as an indicator of the level of moral hazard of an entrepreneur which is dependent upon the equity share, the expected profit, the capital of the project and debt (if exists).

The index MHI is inversely proportional to *nc* and proportional to the probability of default so it can be use as an index for the moral risk faced by the investor. We will use the notations MHI_e and MHI_D for m

oral hazard indices for *mudarabah* and *murabahah* respectively.

Example of MHI calculation

	F	E(x)	α	D(Markup) Bank Profit	β Entrepreneur Profit	Bank Equity share	MHI
Mudarabah	15000	50000	0.4	0	14000	21000	1.03
	15000	40000	0.4	0	10000	15000	1.2
	15000	30000	0.4	0	6000	9000	1.42
Murabahah	15000	50000	1	21000	14000	0	1.03
	15000	40000	1	15000	10000	0	1.2
	15000	30000	1	9000	6000	0	1.42

In this example the MHI of mudarabah and murabahah cases are equal since the figures chosen are such that $D = (1 - a)(E - F)$ as stated in Preposition 1 below.

In the above example if $a > 0.4$ then $MHI_e > MHI_D$ and if $a < 0.4$ then $MHI_e < MHI_D$.

2.3 Properties of Moral Hazard Index

$MHI_D > MHI_e$ if E decreases while all other variables remain the same.

$MHI_e > MHI_D$ if E increases while all other variables remain the same

MHI_D is negatives even though the project is profitable (E - F > 0) if E - F - D < 0. In this case the project is not feasible.

MHI_e is always positive if the project is profitable (E - F > 0)

Proposition 1:

The cut-off level of moral hazard for murabahah is equal to the cut-off level of moral hazard for mudarabah if $D = (1 - a)(E - F)$,

Proof :

$$c_D = \frac{E - F - (E - F)(1 - a)}{nF} = \frac{E - F - E + F + a(E - F)}{nF} = \frac{a(E - F)}{nF} = c_e$$

Proposition 2 : MHI_D will always be higher than MHI_e given the same set of E, F *if* $D > (1 - a)(E - F)$

Proposition 3 : MHI_D and MHI_e will always increase if the expected income, E decreases and will decrease if E increase for a any given set of a, n, D, E and F .

Proof :

$$MHI_D = \frac{2F}{\beta(x) + F} = \frac{2F}{E - F - D + F} = \frac{2F}{E - D}$$, so if E increases MHID decreases.

$$MHI_e = \frac{2F}{\beta(x) + F} = \frac{2F}{\alpha(E - F) + F} = \frac{2F}{\alpha E + (1 - \alpha)F}$$, so if E increases MHIe decreases.

Proposition 4 : Rate of change of c_D with respect to E is always higher than the rate of change of c_e for any given set of α, D, E and F .

Proof :

The cut-off level of moral hazard index for *murabahah* is $c_D = \frac{(E - F - D)}{nF}$, and the

cut-off level of moral hazard index for *mudarabah* is $c_e = \alpha \dfrac{(E-F)}{nF}$.

$$\frac{\delta(c_D)}{\delta E} = \frac{\delta\left[\dfrac{(E-F-D)}{nF}\right]}{\delta E} = \frac{1}{nF} \quad \text{and} \quad \frac{\delta(c_e)}{\delta E} = \frac{\delta\left[\dfrac{\alpha(E-F)}{nF}\right]}{\delta E} = \frac{\alpha}{nF}$$

Here $\dfrac{\alpha}{nF} < \dfrac{1}{nF}$ since $0 < \alpha < 1$

Proposition 5 : An entrepreneur has higher probability of defaulting in murabahah contract than in mudarabah contract for any decrease in E due to the moral hazard factor

Proof :

Any decrease in E will decrease the cut-off level of moral hazard (increase probability of default). Since c_D will decrease more than c_e for the same amount of decrease in E then an entrepreneur has higher probability of defaulting in murabahah contract than in mudarabah contract due to the moral hazard factor for any decrease in E.

3 Conclusions

In this paper we propose a simple index to measure the moral hazard of a financial contract offered to an entrepreneur. The index reflex the appropriateness of a loan contract base on the entrepreneur utility function. This index is not in any way intended to be an independent measure of credit risk in itself. It is to be a complement of the existing method in computing the credit risk on corporate loans.

References :

Abel Elizalde (2006), CEMFI Working Paper No. 0606, April 2006

Altma, E. I. and A. Saunders, (1998) "Credit risk measurement: Developments over the last 20 years" Journal of Banking and Finance, vol. 21, pp. 1721 - 1742.

Altman, E. I Haldeman, R.G. and P. Narayanan, (1977) "ZETA Analysis. A new model to identify bankruptcy risk of corporations" Journal of Banking and Finance, vol. 1, pp. 29 - 54.

Anthony Saunders and Linda Allen, (2002)."Credit Risk Measurement, New

Approaches to Value at Risk and Other Paradigms" Second Edition, John Wiley & Sons, Inc. Chapter 2.

Azhar, A.K.M. and Elliott, R. J. R.(2006). On the measurement of product quality in intra-industry trade. Review of World Economics, 142(3), 476-495.

Black, F., and J. C. Cox, (1976) "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," Journal of Finance 31, 351-367.

Chengcheng Hao, Md. Maudod Alam and Kenneth Carling, (2010), "Review of the Literature on Credit Risk Modeling: Development of the past 10 years", Banks and Bank Systems, Volume 5, Issue 3, 2010.

D. E. Allen and R. J. Powell, (2011), "Credit risk measurement methodologies",

19th International Congress on Modelling and Simulation, Perth, Australia, 12-16 December 2011.

Lean Yu, Shouyang Wang, Kin Keung Lai, Ligang Zhou, (2008). "Bio- Inspired Credit Risk Analysis" Springer-Verlag Berlin Heidelberg.

Merton, R., (1974). "On the Pricing of Corporate Debt: the Risk Structure of Interest Rates," Journal of Finance 29, 449-470.

Rajesh K. Aggarwal and Tarik Yousef (2000), "Islamic Banks and Investment Financing". Journal of Money, Credit, and Banking, pp93-120.