Islamic Banking Performance VS its Conventional Counterpart: Using Stochastic Optimal Control Method

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Abstract
Considerable growth of Islamic banking industry during recent years makes it more interesting for researchers, to compare relative performance of Islamic banking system and its conventional counterpart. Although there are several methods for such a comparison, this paper tries, as a new approach, to use stochastic optimal control for this purpose. Stochastic optimal control is a method for finding optimal performance of one economic agent, by maximizing its objective function subject to some stochastic constraints. To construct the optimization problem for banking industry, in this paper, it is assumed that the loan scheme follows the stochastic process of Brownian motion in both Islamic and conventional banking systems. However, since the nature of these two banking systems are basically different, the important characteristics of each system have been considered for its mathematical modeling. One important difference is that in the Islamic banking, in opposite of its conventional counterpart, bank is not the owner of depositor’s money and, just as their Wakil, tries to maximize their profit. The other difference is that in the Islamic banking system there are two broad types of loans: (1) fixed return contracts and (2) Musharakah contracts, while all the loan contracts in the conventional banking system are based on the fixed interest rates. According to the simulation results of the proposed models in this paper, although banking rates are important values for determining the performance of the banking systems, in almost similar situations, Islamic banking system over performs its conventional counterpart.

Keywords: Islamic Banking, Conventional Banking, Stochastic Optimal Control, Brownian Motion.

1. Introduction
Islamic banking industry, has experienced considerable growth in the recent years which makes it an important rival for conventional banking. Actually, the global Islamic banking assets, has grown to 1.3 trillion dollars in 2012 and it is forecasted to grow beyond the milestone of $2 trillion by 2014 (Ernst and Young, 2013). The expansion of Islamic banking industry has gone beyond the border of Islamic countries and deeply entered in to other countries where Muslims are a minority, such as the United Kingdom or Japan (Solé, 2008). This considerable growth caused many scholars and researchers to conduct some analysis about the relative performance of Islamic and conventional banking systems. Ahmad and Hassan (2007) after introducing the concept of Riba and some reasons for its prohibitions in Islam, provide a conceptual comparison between the Islamic and conventional banking systems. According to their study, the Islamic banking system works more efficient than its conventional counterpart since Islamic banking priority is public interests but conventional bank considers its own interests. Moreover, while Islamic banks choose the most efficient projects by exact and multi aspect evaluations, conventional banks just consider credit-worthiness of their clients.

Also some Islamic economic scholars have used real data analysis for such a comparison. Mohamad, Hassan and Bader (2008) examined the cost and profit efficiency for a sample of 80 banks including 37 conventional and 43 Islamic, using the stochastic frontier approach. Also recently, Iqbal (2011) has chosen a sample of twelve Islamic and twelve conventional banks during 1990 to 1998 and tried to calculate and compare some banking industry ratios such as liquidity and capital asset ratios for both systems. In a similar study, Ahmad and Abdul Rahman (2012) have also tried to compare the efficiency of Islamic and conventional banking system in Malaysia using data envelopment analysis. However, none of the above mentioned authors have paid enough attention to the mathematical modelling of all aspects of the banking performance to do a comparative study.

In this paper we try to propose a new approach for comparing the performance of Islamic and conventional banking systems using stochastic optimal control method. Actually, the goal of stochastic optimal control method (or dynamic stochastic optimization) is the optimization of an objective function subject to some constraints of the form of stochastic differential equations. Applying the concept of stochastic process in economic and finance fields goes back to the famous work of Black and Scholes (1973) about option pricing. Afterward, Dangl and Lehar (2004) used a
continuous time stochastic model to find the bank optimal portfolio and its capital requirements. In another study, Mukuddem-Peterson and Peterson (2006) used stochastic optimal control method to minimize market and capital adequacy risk of the bank and suggest one optimal portfolio for the bank loans. Also one can refer to Sheldon (2006) for a review of other applications of stochastic optimal control in economic researches during this period. Later, Mukuddem-Petersen et al. (2007) used stochastic optimal control method for maximizing discounted utility of depository consumption subject to stochastic changes of bank profit over time. In their study they assumed that the loan value provided to customers by the bank is a stochastic random variable which follows a geometric Brownian motion process. Also recently, Abutaleb and Hamad (2012) used this method to derive a simple formula for the optimal level of international reserves in Egypt. However, it should be noticed that the application of stochastic optimal control method for mathematical modelling of Islamic banking system, is a completely new approach which will be addressed in this paper.

To use the stochastic optimal control method for the comparative study of the Islamic and conventional banking industries, first we try to construct different objective functions along with different stochastic constraints for each of these systems. Here, we have to consider the fact that the Islamic banking system and its conventional counterpart are different in both their goals and their methods of operation. The most fundamental difference is that in the Islamic banking system, the bank is not the owner of the deposits and just, as a Wakil for depositors, tries to maximize their profit. The other main difference is the existence of both Musharakah and fixed return contracts in the loan portfolio of the Islamic banks while all the loan contracts in the conventional banking system are based on the fixed interest rates. After constructing the optimization problems for each banking system according to its characteristics, In the next step, we try to solve these stochastic optimization problems using their Hamilton-Jacobi-Bellman equations. This method leads to a stochastic differential equation for each system, which its mathematical solution is hardly possible. So, we have applied Euler-Maruyama Simulation method to obtain the optimal path for key banking variables and compare the performance of Islamic and conventional banking systems assuming different model parameters.

The remainder of this paper is organized as follows: In Section 2, we will try to construct the optimization problem for conventional and Islamic banking systems, by defining the objective functions and stochastic constraints in each system. In Section 3, after introducing Euler-Maruyama Simulation method, we will use this method for simulating the stochastic differential equations, derived for each banking system. Also in this section, assuming different parameter values, we will compare the performance of Islamic and conventional banking systems in different situations. Finally some concluding remarks are given in Section 4.

2. Optimization in the Banking Industry

2.1 Conventional Banking

In the conventional banking system, banks, as financial intermediaries, use their own capital in line with the depositor's resources to provide loans to their customers. However, banks should always consider some restrictions in their operation due to the requirements of supervisory institutions. For example, they have to keep some amount of the depositor's resources as the legal reserve in the central bank account and also some amount of these resources should be kept in more liquid assets such as bonds or treasuries. For a typical bank at time $t$, let we indicate the bank capital by $C_t$, the bank deposits by $D_t$, the bank loans by $L_t$, the bonds and treasuries of the bank by $T_t$ and all of the bank legal reserve in the central bank account by $R_t$; hence, we could write the main banking formula as follows:

$$C_t + D_t = L_t + T_t + R_t$$  \hspace{1cm} (1)

The above equation is derived from the bank balance sheet which shows that the bank capital plus the bank deposits, as its liabilities, should be equal to sum of its different types of assets at time $t$. If we consider the filtered probability space $(\Omega, F, F_t \geq 0, P)$, we could introduce loan process as a standard Brownian motion of the form:

$$dL_t = (C_t + D_t - T_t - R_t)dt + \sigma_t L_t dW_t$$  \hspace{1cm} (2)

where $\sigma_t$ is the volatility of loan process and $W_t$ is a standard Brownian motion with respect to a filtration, $F_t \geq 0$, of the probability space $(\Omega, F, F_t \geq 0, P)$. The assumption of Brownian motion for the bank loan process, also has been used in Mukuddem-Petersen et al. (2007), but those authors, have chosen a geometric Brownian motion of the following form:

$$dL_t = L_t[(r^L_t - c)dt + \sigma L_t dW_t]$$

where $r^L_t$ is the interest rate of the loan and $c$ is its marginal cost. Although mathematically this equation is correct, but it seems to have weak economic logic.
While \( r^L - c \) is the rate of income on loan, why should the amount of bank loans grow at the average rate of this rate? So, in this paper, we have proposed equation 2 for the bank loan stochastic process which its average change over time is based on the balance sheet equation (i.e. equation 1).

The income of this typical bank, at time \( t \), comes from the interest income of the loans to its customers, \( r^L \), and the interest income from the bond and treasuries it holds, \( r^T \). The bank also has to pay the interest of \( r^D \) to its depositors. So, the net interest income of this bank could be written as follows:

\[
Net\,\,Interest\,\,Income_t = r^L L_t + r^T T_t - r^D D_t
\]

To calculate the bank profit, the bank cost structure should be considered as well. Let the cost structure of the bank at time \( t \) have a quadratic from with respect to its deposits and loans as follows:

\[
Cost\,\,of\,\,the\,\,Bank_t = a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2,
\]

where \((a_0, a_1, a_2, b_1, b_2)\) is the vector of cost structure parameters of the bank. Now we can construct our stochastic optimization problem for the value function, \( V(t, L_t) \), as follows:

\[
\max_{D_t} V(t, L_t) = \int e^{-\rho t} (r^L L_t + r^T T_t - r^D D_t - (a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2))dt
\]

\[
\text{s.t.}\,\,dL_t = (\theta L_t - T_t - R_t)dt + \sigma L_t dW_t
\]

where \( \rho \) is the discount rate used for calculating the current value of the bank future profits. It should be noticed that in the above stochastic optimization problem, \( L_t \) is the state variable and \( D_t \) is the control variable so that the value function should be maximized with respect to \( D_t \) subject to the stochastic constraint for \( L_t \). To make the proposed model in equation 3 analytically simpler, we could consider the following equations for \( C_t, T_t \) and \( R_t \):

\[
C_t = \theta L_t
\]

\[
T_t = \delta D_t
\]

\[
R_t = \gamma D_t
\]

Actually, equation 4 shows that the bank should keep its capital level at time \( t \) equal to a fixed proportion of its loans at the same time. If instead of the loan, we use the bank risk weighted assets, the \( \theta \) proportion is the same as capital adequacy ratio which determined by supervisory institutions. Equation 5 shows liquidity management for the bank at time \( t \), in which the bank have to keep some fixed proportion of its deposits as treasuries and bonds. Finally, according to the equation 6 the legal reserve ratio of the bank at time \( t \) is equal to \( \gamma \).

Considering these restrictions, we could rewrite our stochastic optimization problem as follows:

\[
\max_{D_t} V(t, L_t) = \int e^{-\rho t} (r^L L_t + (r^T \delta - r^D) D_t - (a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2))dt
\]

\[
\text{s.t.}\,\,dL_t = (\theta L_t + (1 - \delta - \gamma) D_t)dt + \sigma L_t dW_t
\]

As Hanson (2007) presented in his work, we could solve the above optimization problem through solving the following Hamilton – Jacobi – Bellman equation:

\[
- V_t' = \max_{D_t} \{ e^{-\rho t} [r^L L_t + (r^T \delta - r^D) D_t - (a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2)]
\]

\[
+ V'_L (\theta L_t + (1 - \delta - \gamma) D_t) + \frac{1}{2} V''_L \sigma^2 L_t^2 \}
\]

where \( V_t' \) denotes the first derivative of the value function with respect to \( t \) and \( V'_L \) and \( V''_L \) are first and second derivatives of the value function with respect to \( L_t \), respectively. For simplicity we assume that the volatility of loan process, \( \sigma \), is independent of time. Taking first derivative with respect to \( D_t \) from both sides of equation 8 leads to the optimal value of \( D_t \) as follows:

\[
D_t = \frac{V'_L N e^{\rho t} + (M - a_0)}{2a_2}
\]

where \( N = (1 - \delta - \gamma) \) and \( M = (r^T \delta - r^D) \). Substituting this optimal value of \( D_t \) in equation 8 and finding the form of the value function, \( V(t, L_t) \), we could reach to the optimal value of \( D_t \) as a function of \( L_t \) as follows (see the appendix for more details):

\[
D_t = \frac{NA_1 + (M - a_0)}{2a_2} + \frac{NA_2 L_t}{a_2}
\]

where \( A_1 \) and \( A_2 \) are two parameters with the following values:

\[
A_1 = \frac{a_1 (r^L - b_0) + a_2 N (M - a_0)}{a_2 (\beta - \theta) - A_2 N^2},
\]

\[
A_2 = \frac{a_2 N^2}{\beta - \theta - A_2 N^2}
\]
Solving the above stochastic differential equation leads to the optimal path of loan performance in the conventional banking system. Since the mathematical solution of this stochastic differential equation is hardly possible, we will use simulation methods to solve this equation in Section 3.

2.2 Islamic Banking

For constructing the optimization problem in the Islamic banking system, we have to consider main characteristics of this banking system. The most important characteristic of the Islamic banking system is the prohibition of the Riba concept in the banking operation. Riba has two broad types: (1) Riba Al-Nasia and (2) Riba Al-Fadl, but only the first one is related to the banking operation. What is prohibited by Shariah as Riba Al-Nasia is borrowing with the condition of paying back some additional benefit. This concept of Riba exists when Qard contract exists and the definition of Qard in Islamic jurisprudence is “Transferring the ownership of an asset to another with the grantee that he/she would pay it back” (Iravani, 1999).

So, for eliminating Riba from Islamic banking system, there should be no Qard contract both in getting deposits from customers and providing loan to them. Actually, when depositors put their money in their accounts in one Islamic bank, no ownership transfer would happen and that Islamic bank just try to work with the depositor’s money, as their Wakil, to pay each depositors her share from the gained profit in the banking operation. Also, for providing loan to customers the Islamic bank should use either Musharakah contracts or fixed return contracts like Murabahah to be away from Riba based loan system. Since the bank in Islamic system is nothing but a Wakil for depositors, it seems that in the Islamic banking system the main goal is to maximize the depositor’s profit, in contrast with what we see in the conventional banking system where the bank tries to maximize its own profit.

Considering these differences, in this section, we try to construct a stochastic optimization problem which is compatible with Islamic banking system. Using the same assumption about probability space and also the same notation as the previous section, we could rewrite equation 2 for the stochastic change of loans in the Islamic banking system as follows:

\[ dl_t = (D_t - T_t - R_t) dt + \sigma_t L_t dW_t \]  (12)

What we have presented here in equation 12 for the stochastic changes of the loan process in Islamic banking has two main differences with what we have proposed in equation 2 for conventional banking system. First, there is no bank capital in equation 12, because in Islamic banking system, the bank is just a Wakil for depositors and so its resources also should be based only on depositor’s money. Second, since there are no bonds or treasuries in the Islamic banking model, \( T_t \) in equation 12 is the amount of money that the bank allocates for buying some kinds of Islamic securities like Sokuk. Again, for simplicity, we assume that both \( T_t \) and \( R_t \) are determined as a fixed proportion of \( D_t \) as follows:

\[ T_t = \delta D_t \]
\[ R_t = \gamma D_t \]

Considering the fact that the Islamic bank is a Wakil for depositors and so should maximize their profit, we now introduce the sources of income that are available for the Islamic bank. First of all, via purchasing Sokuk, Islamic bank could earn fixed income of \( r_F \). The other sources of income are based on providing loan to customers. Here, we assume that the Islamic bank invests \( \alpha \) percent of its loans to fixed return contracts like Murabahah and \( (1 - \alpha) \) percent to Musharakah contracts. For fixed return contracts, the Islamic bank should subtract its fee for Wakalah services and pay the remaining amount to the bank depositors. We assume this after fee rate to be denoted by \( r_F \). For Musharakah contracts, we assume that the Islamic bank has one pool of Musharakah projects in different industries, and this portfolio has a minimum expected rate of return. So, the Islamic bank in each sub period pays its depositors this minimum expected rate, after subtracting its Wakalah fee, and at the end of period, when Musharakah contracts are over, calculates the real rate of return of Musharakah portfolio and pays the additional amount of profit to the depositors. Here, we show the minimum expected after fee rate of Musharakah portfolio by \( r_M \) and the real end of period Musharakah portfolio rate of return by \( r_R \).
After these explanations about the income sources in Islamic banking system, and using the same assumption for the cost structure of the this bank as the previous section, we could define the stochastic optimization problem of the Islamic banking system as follows:

\[
\max_{D_t} \quad V(t, L_t) = \int_t^T e^{-\beta t} \left( (\alpha r^F L_t + (1-\alpha) r^M L_t + \delta r^T D_t \right) dt \\
- (a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2)) dt \\
+ (1-\alpha)e^{-\beta T} (r^F - r^M) L_T \\
st. \quad dL_t = ((1-\delta - \gamma) D_t) dt + \sigma L_t dW_t.
\]

To solve the above optimization problem, we construct the following Hamilton–Jacobi–Bellman equation:

\[
-\dot{V}_t = Max \{ e^{-\beta t} [(\alpha r^F L_t + (1-\alpha) r^M L_t + \delta r^T D_t - (a_0 + a_1 D_t + a_2 D_t^2 + b_1 L_t + b_2 L_t^2)], \\
+ V_L (\theta L_t + (1-\delta - \gamma) D_t) + \frac{1}{2} V_{L^2} \sigma^2 L_t^2 \} 
\]

(14)

where, for simplicity, again we assume that the volatility of the bank loan is independent of time. In addition to the above equation, also we have to consider the terminal value function condition. Since we have assumed that in the Islamic banking model, the bank will calculate the actual profit of its Musharakah portfolio and pay the additional profit to depositors, the following condition appears here in the Islamic banking model:

\[
V(T, L_T) = (1-\alpha)e^{-\beta T} (r^F - r^M) L_T
\]

(15)

After taking derivative with respect to \(D_t\) from the equation 14, we could find the optimal value for \(D_t\) as follows:

\[
D_t = \frac{V_{L_t} Ne^{\beta t} + (M - a_t)}{2a_2}
\]

(16)

where \(N = (1-\delta - \gamma)\) and \(M = (r^T \delta)\). Comparing equation 9 and equation 16 shows that the optimal value in the Islamic banking system looks like that of conventional system except for the value of \(M\) which is different in these two systems and would make the comparison of the results for these two banking systems easier. The same calculation as previous section is needed for finding the form of the value function, \(V(t, L_t)\), where the following relationship for \(D_t\) and \(L_t\) would be obtained:

\[
D_t = \frac{N A_t + (M - a_t)}{2a_2} + \frac{N^2 A_t}{a_2} L_t
\]

(17)

where \(A_t\) and \(A_{L_t}\) in the Islamic banking are somehow different from those of conventional banking model. Actually, these parameters would be obtained as follows:

\[
A_t = a_2 (\alpha r^F + (1-\alpha) r^M - b_t) + A_{L_t} N (M - a_t) \\
A_{L_t} = \frac{a_t \beta - A_{L_t} N^2}{a_2} \\
A_{L_t} = \frac{a_2 [(\sigma^2 - \beta) + \frac{4b^2 N^2}{a_2}] - (\sigma^2 - \beta)]}{2N^2}
\]

Substituting equation 17 in the constraint of equation 13, we reach to the following stochastic differential equation for the Islamic banking loan performance:

\[
dL_t = \frac{N^2 A_t + N (M - a_t)}{2a_2} + \frac{N^2 A_{L_t}}{a_2} L_t dt + \sigma L_t dW_t
\]

(18)

Assuming the form of the value function to be known, we could use the terminal condition of equation 15 to find the relationship between the proportion of Musharakah contracts in the bank loan portfolio, \((1-\alpha)\), and actual additional profit of these contracts, \((r^F - r^M)\), as follows:

\[
r^F - r^M = \frac{V(T, L_T)}{(1-\alpha)L_T}
\]

(19)

As we will see in the next section, equation 19 could help us to determine the optimal proportion for Musharakah contracts. According to this equation, when we dedicate more proportion of loans to the Musharakah contracts, we need less compensation as actual profit; but when we invest low proportion of loans in Musharakah contracts, we have to use these resources in more profitable projects in such a way that we could pay more additional profit to the depositors.
3. Simulation of the Banking Performance

3.1 Euler-Maruyama Simulation method

Since finding the exact solutions for the stochastic differential equations of the loan performances in the conventional and Islamic banking industries is hardly possible, we try here to use the Euler-Maruyama simulation method for solving these equations and finding the optimal paths of loan and deposit in the two banking systems.

Hanson (2007) has used Euler-Maruyama simulation method to find a numerical solution for the following stochastic differential equation:

$$dX_t = f(X_t, t)dt + g(X_t, t)dW_t.$$

In this method, one can derive the increment of $X$ from time $k$ to time $k+1$ as follows:

$$dX_k = X_{k+1} - X_k = f(X_k, t_k)\Delta t + g(X_k, t_k)\Delta W_k$$

where $\Delta t$ is a small time step and $\Delta W_k = W(t_k + 1) - W(t_k)$. Now, if we simulate $\Delta W_k$ process via a series of normal random variables with zero mean and $\Delta t^2$ variance we can generate $dX_t$ according to equation 20. Actually, if we have the value of $X$ at the time $t_0$ and the simulated $dX_t$ values, we could generate all the other states of $X_t$ process over our favorite period of time.

In the next section, we will use the same method for simulating stochastic differential equations obtained in the previous section, for both conventional and Islamic banking systems. For that, we take the initial value of $L_0$ equal to one and also we consider 6 months for time period; it means that our optimization problem and its related simulation occurs for $t = 1, ..., 6$. To have a precise approximation of $\Delta t$, we divide our simulation period to 1000 small sub-periods.

3.2 Comparing Islamic and Conventional Banking Performance

Using simulation methods for solving equations 11 and 18 leads us to the optimal path of $L_t$ in conventional and Islamic banking systems, respectively. Also substituting the value of $L_t$ in equation 10 and equation 17, we could find the optimal value of $D_t$ over time for both banking systems. We have three types of parameters in our model: (1) Deposit and loan rates, (2) Cost structure parameters and (3) Policy making parameters. These parameters and their hypothetical values are presented in Table 1.

### Table 1: Model Parameters; descriptions and values

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Banking System</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Volatility of Loan</td>
<td>both Systems</td>
<td>0.20</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount Rate</td>
<td>both Systems</td>
<td>0.04</td>
</tr>
<tr>
<td>$r^L$</td>
<td>Rate of Loans</td>
<td>Conventional</td>
<td>0.07</td>
</tr>
<tr>
<td>$r^D$</td>
<td>Rate of Deposits</td>
<td>Conventional</td>
<td>0.06</td>
</tr>
<tr>
<td>$r^M$</td>
<td>Expected Rate of Musharakah Contracts (Minus bank fce)</td>
<td>Islamic</td>
<td>0.07</td>
</tr>
<tr>
<td>$r^F$</td>
<td>Rate of Fixed Return Contracts (Minus bank fce)</td>
<td>Islamic</td>
<td>0.06</td>
</tr>
<tr>
<td>$r^T$</td>
<td>Rate of Securities</td>
<td>both Systems</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Cost structure parameters**

- $a_0$: Drift or Fixed Cost (both Systems: 0.00)
- $a_1$: Coefficient of $L_t$ (both Systems: 0.05)
- $a_2$: Coefficient of $L_t^2$ (both Systems: 0.05)
- $b_1$: Coefficient of $D_t$ (both Systems: 0.05)
- $b_2$: Coefficient of $D_t^2$ (both Systems: 0.05)

**Policy making parameters**

- $\theta$: Proportion of $L_t$ that must hold as Capital (Conventional: 0.08)
- $\alpha$: Share of Fixed Return Contracts from total Loans (Islamic: 0.70)
- $\delta$: Proportion of $D_t$ that holds as Securities (both Systems: 0.10)
- $\gamma$: Proportion of $D_t$ that holds as Reserve (both Systems: 0.10)

Considering parameters and their initial values which are given in Table 1, we could calculate the coefficients in the stochastic differential equations of 11 and 18. In the next step we use Matlab software for simulation of these equations to derive the optimal path of loan variable, $L_t$, during our desirable period. Figure 1 shows the results of such a simulation. As we could see in this figure, under the assumptions that were adopted in Table 1, loan process in the Islamic banking model over performed the loan process of the conventional banking model which shows that, in
almost similar situations, Islamic banking system is more capable for providing loans to its customers. Similarly, Substituting the value of $L_t$ for conventional and Islamic banking system in equation 10 and equation 17 respectively, we could also find the optimal value of $D_t$ over time. In Figure 2 we could compare the performance of deposits in these two banking systems.

![Diagram](image1)

**Figure 1:** Conventional and Islamic Banking Loan Path: Simulation Results

It should be considered that changing the assumed parameter values in Table 1 would lead us to different results. Doing Matlab simulations with different values for the parameters show that the comparative results between the two banking systems is not affected by changing cost structure parameters. What actually affect the relative performance of Islamic and conventional banking system is the change in rate's levels. If the increase or decrease in the model rates is assumed to be equal in both systems, again there is no change in the results. But, if we decline deposit and loan's interest rate in conventional banking system (i.e., $r^D$ and $r^L$) for example by two percents or increase the after fee rates of fixed return contracts and expected Musharakah contracts in an Islamic banking system (i.e., $r^F$ and $r^M$) by two percents, the results will change and the performance of conventional banking system would be better than Islamic one. The results of such simulations are shown in Figures 3 and 4, respectively.

![Diagram](image2)

**Figure 2:** Conventional and Islamic Banking Deposit Path: Simulation Results

![Diagram](image3)

**Figure 3:** Simulation Results: $r^D = 0.04$ and $r^L = 0.05$; Other parameters similar to the values in Table 1
4. Conclusion

Comparing Islamic banking performance with its conventional counterpart is one of the main interests of scholars in recent years, but using stochastic optimal control method for this comparison is a new approach which is applied in this paper, for the first time. Actually, we have assumed that providing loan to customers in the banking industry follows the Brownian motion process. To construct the stochastic optimization problems for Islamic and conventional banking, an objective function and a stochastic constraint has been considered for each system. Obviously, since Islamic banking industry has its own characteristics, we have to differentiate both objective function and constraint of this banking system from conventional ones. The two main characteristics of Islamic banking system, which cause differences in its optimization problem are, (1) having two types of loan, Musharakah and fixed return contracts and (2) playing the role of Wakil for the depositors and trying to maximize their profit. Solving the stochastic optimization problems of these banking systems, lead us to a stochastic differential equation for each system and since the mathematical solution of such equations are hardly possible, we have used simulation methods for numerical solutions.

Comparing the simulation results shows that, first of all, the most important parameters for banking systems to run better are their relative rates. This shows the importance of competition between Islamic and conventional banking systems i.e. each banking systems that offers lower rates to its customers, when all other things are equal, over performs the other system. Second, when the rates of two systems are almost the same, Islamic banking performance seems better than conventional banking.
system and, third, by allocating more portion of loan portfolio to Musharakah contracts, Islamic bank is less worried about the profitability of Musharakah projects since it has to make less real calculated end of period compensation to its depositors.

APPENDIX: Derivation of the Value Function

Since the derivation method of the value function is the same for both conventional and Islamic banking systems, here we just explain the steps of such a derivation for conventional banking system. Substituting the optimal value of \( D_t \), as shown in equation 9, in the Hamilton – Jacobi – Bellman equation of equation 8, and assuming \( N = (1 - \delta - \gamma) \) and \( M = (r^T \delta - r^D) \), we could write the following equation:

\[
V'(t) = e^{-\beta t} r^l L_t + V'_t M N + e^{-\beta t} M (M - a_t) - a_0 e^{-\beta t} - a_t e^{-\beta t} (M - a_t)
\]

\[
V''_t N^2 + \frac{a_0 e^{-\beta t} (M - a_t)^2}{2a_2} - \frac{a_t e^{-\beta t} (M - a_t)^2}{2a_2} - b_1 e^{-\beta t} L_t - b_2 e^{-\beta t} L_t^2 + V'_t \theta L_t
\]

\[
= V'_t N^2 e^{-\beta t} + \frac{V'_t M (M - a_t)}{2a_2} + \frac{V'_t (M - a_t)}{2a_2} - b_1 e^{-\beta t} L_t - b_2 e^{-\beta t} L_t^2 + V'_t \theta L_t
\]

where \( V'_t \) is the first derivative of value function with respect to \( t \) and \( V''_t \) and \( V'_{L_t} \) are first and second derivatives of the value function with respect to \( L_t \), respectively. After some arrangement and factorization we reach to the following equation:

\[
V'(t) = \left( \frac{(M - a_t)^2}{2a_2} - a_0 \right) e^{-\beta t} + \frac{N (M - a_t)}{2a_2} \theta L_t + \frac{e^{\beta t} N^2}{2a_2} V''_t
\]

\[
+ \frac{1}{2} \frac{V'_t \sigma^2 L_t^2 + (r^l - b_1) e^{-\beta t} L_t - b_2 e^{-\beta t} L_t^2}{2a_2}.
\]

Looking at the above simplified equation, we could guess that the value function should have a quadratic term with respect to \( L_t \) and an exponential term with respect to \( t \) as follows:

\[
V(t, L_t) = (A_0 + A_1 L_t + A_2 L_t^2) e^{-\beta t} + c,
\]

where \( c \) is a constant parameter. Now, for finding the parameters \( A_0, A_1 \) and \( A_2 \), first we should calculate the derivatives of the above guessed value function:

\[
V'_t = -\beta (A_0 + A_1 L_t + A_2 L_t^2) e^{-\beta t}
\]

\[
V''_t = (A_1 + 2A_2 L_t) e^{-\beta t}
\]

\[
V''_t = 2A_2 e^{-\beta t},
\]

then we should substitute these derivatives in equation 21 and equalize coefficients of both sides to reach to the following parameter values:

\[
A_0 = \frac{1}{A_0} \left( \frac{(M - a_t)^2 + N^2 A_1^2}{4a_2} + A_1 N (M - a_t) - a_0 \right)
\]

\[
A_1 = \frac{a_2 (r^l - b_1) + A_2 N (M - a_t)}{a_2 (\beta - \theta) - A_2 N^2}
\]

\[
A_2 = \frac{a_2 \left[ \frac{2(\theta - \beta + \sigma^2)^2 + 4b_2 N^2}{a_2} \right] - (2\theta - \beta + \sigma^2)}{2N^2}
\]

References


1. Introduction

Unlike Islamic banking which is understood by majority of the market players or included in the global banking system, the development of Takaful still requires practical innovation and Shariah evidence to solve some issues due to jurisdictionally differences in terms of its product structures and market practices. There are reasons

2. Concept of Tabarru Commitment in Takaful Undertakings

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Abstract

Takaful undertaking is one of the developing and growing areas of Islamic finance in the contemporary world. It is a Shariah-justified scheme adapted in the economy as an alternative to the conventional insurance. In 1985, the Islamic Fiqh Academy, an organization of Muslim experts from Islamic countries, resolved that conventional insurance is haram (forbidden) because it contains element of gharar (uncertainty) and riba (usury). However, the Academy approved the use of mutual form of insurance that conforms with the principles of the Shariah as an alternative to conventional insurance. This decision spurred the enormous growth of the takaful industry. Therefore, this paper will focus on the basis of the legality of takaful undertaking, the meaning of gharar and when it is forgivable in takaful industries. It will also explain and analyze how the concept of tabarru commitment in takaful undertaking is related to contractual arrangement, which is considered as the best practice in the takaful operation. These contractual rules distinguish takaful undertaking from the conventional insurance and various aspects of underwriting surplus and investment activities. This research will conclude by highlighting some regulatory considerations where takaful fund will be treated as separate legal entity and independent financial liability.

Keywords: Takaful undertaking, Tabarru commitment, Contributions, Surplus, Gharar

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